Dynamics of Multibody Systems Final Examination
(Graduate School)

Take-Home Exam's deadline: 2019/1/9 Wednesday 3:00PM

1. Show $(\tilde{\mathbf{v}})^2$ is symmetric. (15%)

2. $\bar{\mathbf{r}} = \begin{bmatrix} 0 & 1 & 5 \end{bmatrix}^{\mathrm{T}}$, $\mathbf{v}_a = \begin{bmatrix} 1 & 0 & 3 \end{bmatrix}^{\mathrm{T}}$, **and** $\dot{\theta} = 20$ **rad/sec. Find** $\boldsymbol{\omega}$ **and A at time t = 0.1 sec. Find** $\dot{\mathbf{r}}$ **at time t = 0.2 sec of the point** $\bar{\mathbf{r}}$ **in this problem. (15%)**

3. Initially, the direction of the axes of a body *i* are given in global coordinates as

Direction of $\mathbf{X}_1^i$: $\mathbf{v}_1^i = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}^{\mathrm{T}}$

Direction of $\mathbf{X}_2^i$: $\mathbf{v}_2^i = \begin{bmatrix} \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \end{bmatrix}^{T}$

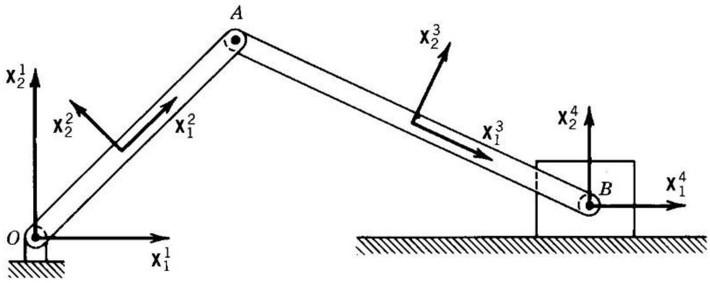
Direction of $\mathbf{X}_3^i$: $\mathbf{v}_3^i = \begin{bmatrix} \frac{-2}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}^{T}$

The body is rotated about 60° about its $\mathbf{X}_3^i$ axis, and then 45° about its $\mathbf{X}_1^i$ axis. Find the transformation matrix that defines its final orientation. (15%)

4. $\mathbf{A} = \begin{bmatrix} 0.6667 & -0.3333 & 0.6667 \\ 0.6667 & 0.6667 & -0.3333 \\ -0.3333 & 0.6667 & 0.6667 \end{bmatrix}$. **(20%)**

(i) Find θ and v that A represents.
(ii) Find the Euler Parameters for the matrix A.
(iii) Find the Rodriguez parameters for the matrix A.
(iv) Find the Euler angles for the matrix A.

5. Find the velocities of the coordinates of the bodies of Crank-Slider Mechanism in terms of R_1^4, the x coordinate of the slider. (15%)



$$\mathbf{q}_r^1 = \begin{bmatrix} R_1^1 & R_2^1 & \theta^1 \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{q}_r^2 = \begin{bmatrix} R_1^2 & R_2^2 & \theta^2 \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{q}_r^3 = \begin{bmatrix} R_1^3 & R_2^3 & \theta^3 \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{q}_r^4 = \begin{bmatrix} R_1^4 & R_2^4 & \theta^4 \end{bmatrix}^{\mathrm{T}}$$

6. A one-element, 2D beam has the shape function

$$\mathbf{S}^i = \begin{bmatrix} \xi & 0 & 0 \\ 0 & 3(\xi)^2 - 2(\xi)^3 & l\left((\xi)^3 - (\xi)^2\right) \end{bmatrix}$$

and generalized coordinates

$$\mathbf{q}^i = \begin{bmatrix} 3.0 & 2.0 & \frac{\pi}{2} & 0.5\times10^{-3} & 10^{-3} & 10^{-5} \end{bmatrix}^{\mathrm{T}}$$

Determine the global position of the points $\xi = 0.5,\ 1.0$**.**

$$\dot{\mathbf{q}}^i = \begin{bmatrix} 0 & 0 & 50 & 5\times10^3 & 10^2 & 3\times10^4 \end{bmatrix}^{\mathrm{T}}$$

and

$$\ddot{\mathbf{q}}^i = \begin{bmatrix} 0 & 0 & 0 & 5\times10^4 & 10^3 & 2.5\times10^5 \end{bmatrix}^{\mathrm{T}}$$

Find the velocities and accelerations of the points $\xi = 0.5,\ 1.0$ **. (15%)**

7. Derive the transformation matrix A and evaluate the transformed vector $\bar{\mathbf{r}} = \begin{bmatrix} 0 & 2 & -6 \end{bmatrix}^{\mathrm{T}}$ **for a 20° rotation about the vector** $\mathbf{a} = \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}^{\mathrm{T}}$ **. (15%)**

PS: Please solve for the above problems and submit the answer papers with the final report to Prof. Y. L. Hwang's office before 2019/1/9 Wednesday 3:00PM.