

Dynamics of Multibody Systems Final Examination (Graduate School)

Take-Home Exam's deadline: 2022/1/16 24:00

1. Show $(\tilde{\mathbf{v}})^2$ is symmetric. (15%)

2. $\bar{\mathbf{r}} = [0 \ 1 \ 5]^T$, $\mathbf{v}_a = [1 \ 0 \ 3]^T$, and $\dot{\theta} = 20$ rad/sec. Find $\boldsymbol{\omega}$ and \mathbf{A} at time $t = 0.1$ sec. Find $\dot{\bar{\mathbf{r}}}$ at time $t = 0.2$ sec of the point $\bar{\mathbf{r}}$ in this problem. (15%)

3. Initially, the direction of the axes of a body i are given in global coordinates as

Direction of \mathbf{X}_1^i : $\mathbf{v}_1^i = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}^T$

Direction of \mathbf{X}_2^i : $\mathbf{v}_2^i = \begin{bmatrix} -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \end{bmatrix}^T$

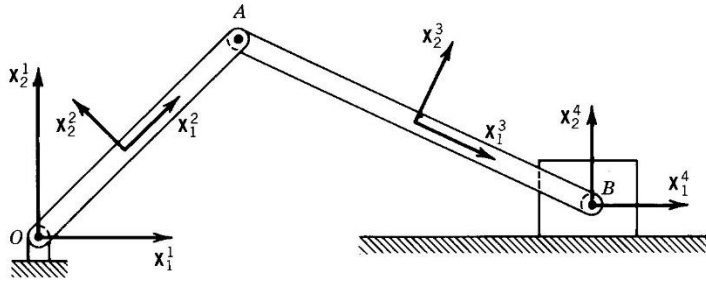
Direction of \mathbf{X}_3^i : $\mathbf{v}_3^i = \begin{bmatrix} -\frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}^T$

The body is rotated about 60° about its \mathbf{X}_3^i axis, and then 45° about its \mathbf{X}_1^i axis. Find the transformation matrix that defines its final orientation. (15%)

4. $\mathbf{A} = \begin{bmatrix} 0.6667 & -0.3333 & 0.6667 \\ 0.6667 & 0.6667 & -0.3333 \\ -0.3333 & 0.6667 & 0.6667 \end{bmatrix}$. (20%)

- (i) Find θ and \mathbf{v} that \mathbf{A} represents.
- (ii) Find the Euler Parameters for the matrix \mathbf{A} .
- (iii) Find the Rodriguez parameters for the matrix \mathbf{A} .
- (iv) Find the Euler angles for the matrix \mathbf{A} .

5. Find the velocities of the coordinates of the bodies of Crank-Slider Mechanism in terms of R_1^4 , the x coordinate of the slider. (15%)



$$\mathbf{q}_r^1 = [R_1^1 \quad R_2^1 \quad \theta^1]^T$$

$$\mathbf{q}_r^2 = [R_1^2 \quad R_2^2 \quad \theta^2]^T$$

$$\mathbf{q}_r^3 = [R_1^3 \quad R_2^3 \quad \theta^3]^T$$

$$\mathbf{q}_r^4 = [R_1^4 \quad R_2^4 \quad \theta^4]^T$$

6. A one-element, 2D beam has the shape function

$$\mathbf{S}^i = \begin{bmatrix} \xi & 0 & 0 \\ 0 & 3(\xi)^2 - 2(\xi)^3 & l((\xi)^3 - (\xi)^2) \end{bmatrix}$$

and generalized coordinates

$$\mathbf{q}^i = \left[3.0 \quad 2.0 \quad \frac{\pi}{2} \quad 0.5 \times 10^{-3} \quad 10^{-3} \quad 10^{-5} \right]^T$$

Determine the global position of the points $\xi = 0.5, 1.0$.

$$\dot{\mathbf{q}}^i = \left[0 \quad 0 \quad 50 \quad 5 \times 10^3 \quad 10^2 \quad 3 \times 10^4 \right]^T$$

and

$$\ddot{\mathbf{q}}^i = \left[0 \quad 0 \quad 0 \quad 5 \times 10^4 \quad 10^3 \quad 2.5 \times 10^5 \right]^T$$

Find the velocities and accelerations of the points $\xi = 0.5, 1.0$. (15%)

7. Derive the transformation matrix A and evaluate the transformed vector

$$\bar{\mathbf{r}} = [0 \quad 2 \quad -6]^T \quad \text{for a } 20^\circ \text{ rotation about the vector } \mathbf{a} = [-2 \quad 1 \quad 3]^T. \text{ (15\%)}$$

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